SAMPLE PAPER - II

SECTION - A (1 mark questions)

1. Solve for x $[1 \quad x]\begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix} = 0$
2. If A and B are symmetric matrices show that AB-BA is skew symmetric matrix
3. Find the value of k if the matrix $\begin{bmatrix} k & 1 \\ 2 & -4 \end{bmatrix}$ is singular
4. If A and B are two events such that P(A)=0.3 ; P(B)=0.2 and P($A \cap B$)=0.05. Are A and B independent events?
5. Given that $P(\overline{A}) = 0.4; P(B) = 0.2$ and $P(A/B) = 0.5$, find $P(A \cap B)$
6. Find the projection of $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ on $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

SECTION - B (4 marks questions)

7. Form the differential equation representing the family of ellipses having foci on the *y*-axis and centre at the origin.

   OR

   Solve the differential equation $\frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x}$

8. Find the general solution of the differential equation $\frac{dy}{dx} + y = 1, y \neq 1$
9. Using properties of determinants prove that

$$\begin{vmatrix} 1 + a^2 - b^2 & 2ab & -2b \\ 2ab & 1 - a^2 + b^2 & 2a \\ 2b & -2a & 1 - a^2 - b^2 \end{vmatrix} = \left(1 + a^2 + b^2\right)^3$$

10. Write in the simplest form $\tan\left[\frac{1}{2}\sin^{-1}\left(\frac{2x}{1+x^2}\right) + \frac{1}{2}\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right]$
11. For any two vectors $\vec{a}$ *and* $\vec{b}$ prove the triangle inequality $\left|\vec{a} + \vec{b}\right| \leq \left|\vec{a}\right| + \left|\vec{b}\right|$ OR If $\vec{a} = \hat{i} + \hat{j} + \hat{k}; and, \vec{b} = \hat{j} - \hat{k}$ then find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}\, and, \vec{a}.\vec{c} = 3$

12. Find the intervals in which the function f given by $f(x)=x^3+\frac{1}{x^3}, x\neq 0$ is (i) increasing (ii) decreasing

13. Differentiate $y=\sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}$ with respect to x

14. Find $\frac{d^2y}{dx^2}, when\ x=a(\cos\theta+\theta\sin\theta), and, y=a(\sin\theta-\theta\cos\theta)$

OR Using Mean Value Theorem find a point on the parabola $y=(x-3)^2$ where the tangent is parallel to the chord joining (3,0) and (5,4)

15. Evaluate $\int\frac{1}{3x^2+13x-10}dx$

16. Evaluate $\int_2^8|x-5|dx$, by using properties of definite integrals

17. Prove that $\int_0^\pi\frac{x}{1+\sin x}dx=\pi$ OR Evaluate $\int\frac{xe^x}{(x+1)^2}dx$

18. An urn contains 4 red and 3 blue balls. Find the probability distribution of the number of blue balls in a random draw of 3 balls, one by one with replacement.

19. A company has two plants to manufacture machines. Plant A manufactures 70%and plant B manufactures 30% machines. At plant A, 80% machines are rated of standard quality and at plant B, 90% machines are rated of standard quality. A machine is chosen at random and is found to be of standard quality. What is the probability that it was manufactured by plant A ?

SECTION - C (6 mark questions)

20. Consider $f:R_+\to[-5,\infty)\ given\ by\ f(x)=9x^2+6x-5$. Show that f is invertible .find the inverse of f.

21. Find the inverse of the matrix $\begin{bmatrix}1&3&-2\\-3&0&-5\\2&5&0\end{bmatrix}$ using elementary row transformation. OR

Using matrices solve the system $2x-3y+5z=11;\ 3x+2y-4z=-5;\ x+y-2z=-3$

22. A jet of an enemy is flying along the curve y= x^2+2.A soldier is placed at the point (3,2) . What is the nearest distance between the soldier and the jet ?

OR An open box with a square base is to be made out of a given quantity of cardboard of area c^2 .Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$

23. Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{\sqrt{2}}$

24. Two go downs A and B have grain capacity of 100 quintals and 50 quintals respectively. They supply to 3 ration shops, D, E and F whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from the godowns to the shops are given in the following table:

Transportation cost per quintal (in Rs)		
From/To	A	B
D	6	4
E	3	2
F	2.50	3

How should the supplies be transported in order that the transportation cost is minimum? What is the minimum cost?

25. Show that the lines $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}; \frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ are coplanar. Also find the equation of the plane containing the lines.

26. Find the equation of the plane which contains the line of intersection of the planes $\vec{r}.(\hat{i}+2\hat{j}+3\hat{k})-4=0$ and $\vec{r}.(2\hat{i}+\hat{j}-\hat{k})+5=0$ and which is perpendicular to the plane $\vec{r}.(5\hat{i}+3\hat{j}-6\hat{k})+8=0$

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