SAMPLE PAPER – IV

SECTION – A

1. Find the value of $\sin\{\pi/3 - \sin^{-1}(-1/2)\}$
2. If f: R→R defined by $f(x) = 3x^2 - 2$, find $f \circ f$
3. Evaluate : $\int \frac{\sec^2(\log x)}{x}dx$

4. If $f(x) = \begin{cases} \frac{\sin 3x}{x} & x \neq 0 \\ k & x = 0 \end{cases}$ is continuous at x = 0, find k.

5. Find a unit vector parallel to the sum of vectors $\vec{a} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

6. Write the direction ratios of the line $\frac{2x-3}{5} = \frac{5-3y}{2} = \frac{3-z}{-3}$

SECTION- B

7. By using properties of determinants, show that

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

8. Consider f: $R^+ \to (-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$ show that f is invertible . Also find f^{-1} .

9. Given that : $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2} & , \text{ if } x < 0 \\ a & , \text{ if } x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+x}-4} & , \text{ if } x > 0 \end{cases}$ If $f(x)$ is continuous at x = 0, find the value of a.

10. If $y = (\tan^{-1} x)^2$, show that $(1+x^2)^2 \frac{d^2y}{dx^2} + 2x(1+x^2)\frac{dy}{dx} = 2$

11. Find the intervals in which the function f given by f(x) = sinx + cos x, $0 \leq x \leq 2\pi$ is strictly increasing or strictly decreasing.

OR

Prove that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1$.

12. Evaluate: $\int \frac{(3\sin\theta - 2)\cos\theta\, d\theta}{(5 - 4\sin\theta - \cos^2\theta)}$ OR

Evaluate $\int \frac{dx}{\sqrt{\sin^3 x \sin(x+a)}}$

13. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c} \cdot \vec{d} = 15$.

14. Find the shortest distance between the lines :

$$\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1} \text{ and } \frac{x-3}{1}=\frac{5-y}{2}=\frac{z-7}{1}$$

15. Solve the differential equation : $\frac{dy}{dx}-3y\cot x=\sin 2x$, given that y = 2 when x = $\pi/2$.

16. A die is thrown twice and the sum of numbers appearing is observed to be 7. What is the conditional probability that the number 2 has appeared at least once.

17. Express $\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$ as a sum of a symmetric and skew symmetric matrices.

18. Solve for x : $\tan^{-1}(x+2)+\tan^{-1}(x-2)=\tan^{-1}\left(\frac{8}{79}\right)$ $(x>0)$

19. Evaluate : $\int_0^{\pi/4}\log(1+\tan x)dx$

Section - C

20. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability accidents is 0.01, 0.03 and 0.15 respectively. One of the insured persons met with an probability that he is a scooter driver? What moral value will you assign to all?

21. Find the equation of the plane that contain that contains the point(1,-1,2) and is perpendicular to each of the planes 2x +3y -2z =5 and x +2y -3z =8.

22. Find the area of the smaller region bounded by the ellipse ;

$4x^2 + 9y^2 = 36$ and the line $2x + 3y = 6$

or

Find the area of the region : $\{(x,y) \,;\, 0 \le y \le x^2 + 1 \,,\, 0 \le y \le x + 1 \,,\, 0 \le x \le 2\}$

23. Find the solution of the differential equation :

$$y - x\frac{dy}{dx} = a\left(y^2 + x^2\frac{dy}{dx}\right) \quad \text{satisfying } x = a\,,\ y = a.$$

24. A window is in the form of rectangle surmounted by a semi-circular opening. Total perimeter of the window is 10m. What will be dimensions of the whole opening to admit maximum light and air?

(i) How does having large windows help us in saving electricity & conserving environment?

(ii) Why optimum use of energy is required in the Indian context?

25. Find the product of $A=\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ *and* $\mathbf{B}=\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and hence solve the system of linear equations :

$$x - y + z = 4$$

$$x - 2y - 2z = 9$$

$$2x + y + 3z = 1$$

26. A merchant plans to sell two types of computers- a desktop model and a portable model that will cost Rs.25000/- and Rs.40000/- respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than 70 lakhs and if his profit on the desktop model is Rs.4500/- and on portable model is Rs.5000/-. Also find the maximum profit.